NON-CAUSALITY AND TRANSREAL NUMBERS: THE WORLD SEEN AT SPEED OF LIGHT.

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In a conference given on February 6, 1939, on the occasion of the James Scott Prize, the English physicist Paul Dirac states the following about the relation between Mathematics and Physics:

Pure mathematics and physics are becoming ever more closely connected, though their methods remain different. One may describe the situation by saying that the mathematician plays a game in which he himself invents the rules while the physicist plays a game in which the rules are provided by Nature, but as time goes on it becomes increasingly evident that the rules which the mathematician finds interesting are the same as those which Nature has chosen. It is difficult to predict what the result of all this will be. Possibly, the two subjects will ultimately unify, every branch of pure mathematics then having its physical application, its importance in physics being proportional to its interest in mathematics..

The famous physicist Dirac, one of the greatest exponents of Contemporary Physics, warns us, in the quote above, to the fact that Theoretical Physics increasingly becomes similar to pure Mathematics. On many occasions, as Dirac himself experimented in his studies on the relativistic character of the Schrodinger Equation for the Electron, a careful analysis of the "unusual or unacceptable" mathematical possibilities for describing Nature, according to a conventional theoretical Physicist who already fixes in advance what Nature allows as its mathematical expression, can lead to physical significant situations *that are completely nonsense to an usual Physicist.* In the case of the "*very unusual*" Dirac, when considering the relativistic

Schrodinger equation for the Electron case, the mathematical possibility of particles with extravagant "negative energies" were not neglected, and from this consideration, the postulate of the existence of antimatter came naturally. In this way, purely mathematical concepts, such as negative numbers, helped to expand our understanding of Nature from the somewhat strange notion, from the usual physical point of view, of "negative energy".

In another excerpt from the conference, Dirac points out that a physicist, when dealing with a given theoretical question, should choose a mathematics that he deems most efficient or appropriate for the issue at hand; and this choice, not by desconsidering pragmatic or utilitarian criteria, could be made guided by the beauty of the chosen mathematics. According to Dirac's words:

The trend of mathematics and physics towards unification provides the physicist with a powerful new method of research into the foundations of his subject, a method which has not yet been applied successfully, but which I feel confident will prove its value in the future. The method is to begin by choosing that branch of mathematics which one thinks will form the basis of the new theory. One should be influenced very much in this choice by considerations of mathematical beauty.

Thus, following Dirac's methodological advice and taking into account the technical limitations of the author of this article (a self-taught in theoretical physics), I propose in this paper to present an *intuitive and philosophical* approach to the concept of *reference-frame* in the Theory of Special Relativity based on the thesis that such a concept must be treated mathematically by three complementary and unquestionably beautiful theories, namely:

1) *The theory of real numbers* (reference-frames that move relatively at speeds lower than that of light);

2) *The theory of complex numbers* (reference-frames that move at speeds greater than that of light);

3) *Theory of transreal numbers* (reference-frames that move relatively with speeds equal to that of light).

These three mathematical ways of approaching the concept of reference-frame, besides being beautiful as already mentioned, show us how the metaphysical notion of Causality behaves in these three types of reference-frame.

In order to understand how the notion of causality can be analyzed mathematically, starting from the mathematical theories mentioned above, let us begin by presenting a somewhat expanded version of the special theory of relativity, an expansion whose aim is to introduce in the phenomenological framework of Einsteinian theory particles that move with speed above the speed of light; and it is from this introduction that the three reference-frames mentioned above emerge. This expansion is not consensual among scholars of special theory of relativity (hereinafter, called STR), since it introduces a non-physical or unobservable world composed of superluminal particles. However, the beauty and heuristic strength that comes from this expansion is unquestionable, and in view of that such expansion will be adopted here as fundamental to a philosophical and metaphysical understanding of the STR.

It is now necessary to present the fundamental postulates of SRT in such a way that no restrictions are made on the relative speed of the particles presupposed in the theory.

In their article "Causality and Tachions in Relativity", the physicists P. Caldirola and E. Recami present the three postulates of the expanded STR (or revisited, according to the denomination used by the aforementioned authors) in the following way:

Even today, the best 'background' for analyzing the essential aspects of time and causality is still that of SR [SRT], in which the framework is a fourdimensional, pseudo-Euclidean space-time. Let us remember that a suitable choice of postulates for the theory of SR [...] is the following [...]:
(I) ***Principle of Relativity****: Physical laws of Electromagnetism and of Mechanics are covariant (= invariant in form) when going from an inertial observer to another inertial observer.*
(2) ***Space-time is homogeneous and space is isotropic****.*
Notice that the postulate of light-speed invariance is not strictly necessary, since it can be derived from the above Postulates (I) and (2). Moreover, if we want, as we do, to avoid information transmission into the past, a Third Postulate is necessary:
(3) ***Principle of Retarded Causality****: For every observer,* **causes** *chronologically precede their own* ***effects****.*

According to such expanded SRT, the *principle of retarded causality* ensures that any observer (reference-frame) assesses his/her *physical world* without "metaphysical extravagances", such as it would happen if we allowed effects to precede their causes in time. Thus, even if an observer is, in relation to a given reference-frame, traveling at a speed greater than that of light, we will be sure that he/she will understand the intelligible structure of reality according to the expected notion of causality: the cause preceding the effect. Obviously, this implies that the clock of this superluminal observer will mark time within an increasing ordered system: *the clock will not travel backwards in time!*

According to the expanded or revisited STR, there are three types of particles that can be considered with their relative velocities as parameters. If we take the particles as inertial reference-frames, then there will be the reference-frames that move with velocities less than that of light in a vacuum (*Bradyons*), equal to that of light *(Luxons*) and the particles that move with velocities greater than the of light

(*Tachyons*). It is worth remembering that the speed of light in a vacuum is equal to 300,000 km / s.

Each particle of the types mentioned above, once taken as an inertial reference frame, is related to the others through Lorentz transformations that will tell us how to make the measurements conversions made in these frames that move in relation to each other.

Thus, given any two reference-frames whose relative movement occurs as a velocity lower than that of light, the Lorentz transformations provide us how each measurement of a given physical quantity will occur in each reference-frame. For example, we may want to evaluate how each of these reference-frames evaluates the movement of a given third observer: if the velocity of that observer is less than the speed of light in relation to the first frame, will this velocity remain lower than that of light in the second frame?

To summarize and generalize questions like that, in such way that one can talk about references-frames that move relatively to each other with velocities equals with light velocity and even greater than velocity of light (reference-frames or observers that are located at an *un-physical world*), I present here some ideas exposed by the physicists V.S. Olkhovsky and E. Reacami (already mentioned in this work) in a work from 1971 for "Lettere al Nuovo Cimento".

The remarks of Olkhovsky and Recami are the following: two reference-frames **R** and **R´** are considered, which have relative velocity equals to **u**. It is also considered that each of these reference-frames is evaluating a particle that moves with velocities **V** and **V’** in relation to **R** and **R´**, respectively. Under such assumptions and according to Lorentz transformation (a general version), the following conditions will hold (in the expressions below, ***c*** is the velocity of light in vacuum):

1) **If u < c , then:**

   If **V < c,** then **V' < c**;

   If **V = c,** then **V' = c;**

   If **V > c,** then **V' > c.**

2) **If u = c, then:**

   If **V < c,** then **V' = c**;

   If **V = c,** then **V' = c;**

   If **V > c,** then **V' = c**

3) **If u > c, then:**

   If **V < c,** then **V' > c**;

   If **V = c,** then **V' = c;**

   If **V > c,** then **V' < c.**

The above relations give us how the movement of a given particle (a Bradyon, a photon or a tachyon) is evaluated by reference-frames that move relatively to each other at subluminal, luminous or superluminal speeds. At first, let us set aside the relative speed equal to that of light (a case that will be carefully studied later) and we will analyze the cases in which the relative speeds are bradyonic or tachyonic.

Reference-frames that move relatively each other at sub-luminal or Bradyonic velocities are considered the cases that actually make up the reference-frames allowed in the physical or observable world; they are the reference-frames that are present in the laboratories and that maintain effective relations of measurements between them; bradyonic reference-frames are the effective observers located in the space-time that constitutes the *physical world.*

From the conditions described above, it is clear that two subluminal or bradyonic observers describe the world of relativistic particles in an equivalent way: the Bradyons, Tachyons or Luxons perceived in **R** remain, respectively, Bradyons, Tachyons or Luxons in **R**'.

When the reference-frames in question move relatively with speeds above the speed of light (tachyon velocities), then the worlds perceived by these reference-frames becomes intriguingly symmetrical: an observed Bradyon in **R** will be a Tachyon in **R** **';** and an observed Tachyon in **R** will become Bradyon for **R**'; and observed Luxons remain Luxons in the transition from **R** to **R'.**

Noteworthy is the fact that reference-frame that move with relative Tachyonic velocities are not considered to be physical ones: they are reference-frames allowed by the expanded **SRT**, but that have only theoretically predicted existence; they are reference-frame that are located in a *non-physical world* and that do not interact causally with the observable physical world, composed exclusively of Bradyonic inertial references.

The mathematical relations that occur between the **R** and **R'** are expressed by the Lorentz Transformations. These transformations relate the measurements made in **R** and **R´.** In particular, considering that the space-time coordinates in **R** are **<x, y, z, t>,** then the corresponding coordinates **<x´, y´. z´, t´>** in **R'** are given by the following equations, assuming the case in which the origins of such reference-frames are coincident and the relative movement between them occurs only in the **x** direction (see the already mentioned article by O. Olkhovsky and E. Recami):

1) $x' = \text{Re}\ [(x + i\xi) - u(t + i\tau)]/\sqrt{1-\beta^2}$

2) $y' = y$

3) $z' = z$

4) $t' = \text{Re}\ [(t + i\tau) - u(x - i\xi)/c^2]/\sqrt{1-\beta^2}$,

in which $\beta = u/c$, and $\beta < 1$ (Bradyonic reference-frames), or $\beta = 1$ (Luxonic "reference-frames"), or $\beta > 1$ (Tachyonic reference-frames).

The expanded Lorentz transformations are functions of complex variables in 1) and 4) expressions. Complex numbers z have the general form

$$z = x + iy$$

in which x is the real component of **z** and y (a real number) is the imaginary component of **z.** In the Lorentz transformations above, the prefix **Re** indicates that we must only consider the real component of the complex expressions present in 1) and 4) – for tachyonic velocities, $\beta > 1$, in virtue of the fact that $\sqrt{1-\beta^2}$ is negative, such expressions are complex numbers of which we must only take into account their real components.

Let us now try to give some phenomenological understanding of Lorentz transformations, in such a way that to the Bradyons we associate real numbers as their *rulers par excellence,* and to the Tachyons we associate also real numbers that come from complex numbers.

First, let's suppose that **R** measures the duration of some event E within his/her surrounding space-time and finds as a result c, a real number. Let us now suppose that a reference frame **R'** which moves with velocity u, $u < c$, relative to **R** , measures the same event E within his/her surrounding space-time; **R'** finds as a result another real number c_1.

And if we now assume that the relative speed between **R** and **R´** is greater than that of light, what can we conceive of as the "phenomenological" view that **R'** has of his/her surrounding world?

In order to carry out this task of answering that question , I will use a methodology that is inspired by the *Dirac´s attitude*, mentioned in the beginning of this draft, regarding the relation between Mathematics and Physics; and such methodology, which I will call "Phenomenological Pythagoreanism", considers that the mathematical language, even if it's not perfectly in tune with what is expected from measurable physical experience, "reveals authentic phenomenological situations, even if extravagant".

Then, according to the "Phenomenological Pythagoreanism", the superluminal reference-frame **R´** sees that a given event ***E***, that has its duration measured in **R** with real numbers, has its duration in **R'** measured with real numbers too! So we can affirm that **subluminal** and **superluminal** observers (*a non-physical observer*) will evaluate the world around them in a similar way regarding their measurements.

From the point of view of metaphysical adequacy to the Principle of Causality, both the reference-frames are in perfect agreement with the already mentioned third postulate of the Theory of Special Relativity expanded.

According to the third postulated of the expanded ***SRT***, it is impossible to exist two events **E1** and **E2**, **E1** being the cause of **E2**, in such way that **E2** precedes **E1**. In other words, the instants when **E2** occurs must be after the instant at which **E1** occurs. In fact, any reference-frame that measures the duration of its events with real numbers satisfies this requirement: real numbers are linearly ordered and, therefore, given two different real numbers ***a*** and ***b*** , it is always valid that ***a < b*** or ***b > a.*.**

Therefore, we can say that authentic reference-frames, which can coordinate the events that are observed from the notion of cause and effect, appear phenomenologically when we have reference-frames that move in relation to each other with bradyonic velocities and when the reference-frames move with respect to each other with tachyonic speeds: *the bradyonic and tachyonic clocks are both calibrated with real numbers.*

Since the bradyonic or tachyonic relative velocities are perfectly acceptable in order to allow the existence of authentic reference-frames, in which expanded ***SRT*** finds its, so to speak, "models" that respect efficient causality (one of the four Aristotelian terms for "Causality" that reminds us of the metaphysical commitment of expanded ***SRT***), what about reference-frames that move relatively at speeds equal to that of light (luxonic reference-frames)? Do they constitute authentic reference-frames? If we consider two inertial reference-frames ***R*** and ***R´*** moving with a relative speed equal to that of light in a vacuum, ***c***, what would ***R´*** see as his/her "measurable world" according to the "Phenomenological Pythagoreanism"? Could ***R´*** coordinate the events in a chain of cause and effect?

In fact, it is something widely known in the special Theory of Relativity that luxonic particles cannot be seen as reference-frames. To exemplify such an impediment, consider the Photon. This particle of zero mass has an intrinsic dynamism and travels through space-time with a speed equal to c, that is, 300,000 km / s. It is a particle that cannot under any circumstances be considered to be at rest and, therefore, does not have its *proper time* according to which it can evaluate the displacements of other bodies.

However, what does *Phenomenological Pythagoreanism* tell us about the possibility of a Luxon being a reference-frame? In other words, what does Mathematics present in Lorentz Transformation about the world seen by an observer who "rides on a luxonic particle"? Does mathematics, as a language that intends to describe the world even in those aspects that are not easily accommodated in the usual physical-mathematical theories (which presuppose that the authentic descriptions of Nature are based solely on the mathematics of the measurable), offer us " metaphysical reasons " for Luxons not to be authentic references?

The idea that I intend to present here is that reference-frames that maintain relative speeds equal to that of light do not constitute legitimate inertial frames, not even in the expanded **SRT**, because in such reference-frames the principle of causality is not verified, since in Luxonic reference-frames it is impossible to speak of an orderly time structure in which events are coordinated. For such an analysis, I will use the mathematics of Transreal Numbers, in which division by zero is perfectly possible, being such a division an anathema for mathematicians trained in the trenches of Real numbers.

Transreal Numbers were created by English computer scientist James Anderson in the late 1990s. Basically, such numbers consist of an extension of the real numbers, in which division by zero is allowed without any contradiction being generated. Obviously, for that, there is a need that the concept of division, a partial recursive function, can be changed and thus "totalized" (something similar occurs with complex numbers, in which the notion of root extraction is expanded in relation to real numbers in such a way as to allow the existence of square roots of negative numbers).

Anderson introduces into real Numbers ℝ three new numbers, namely:

a) $\frac{1}{0} = \infty$ (plus infinity);

b) $-\frac{1}{0} = -\infty$ (minus infinity);

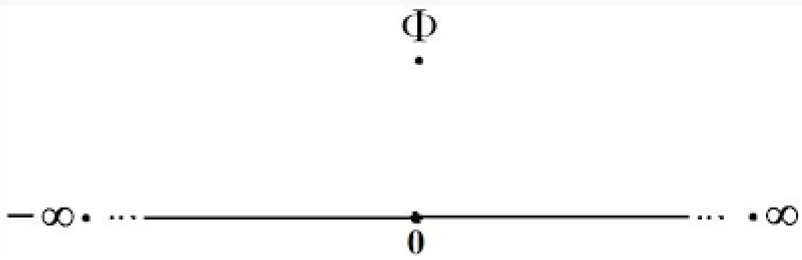
and

c) $\frac{0}{0} = \Phi$ (Nullity).

By means of the addition to these three new numbers to real numbers, we have then the set of Transreal Numbers:

$$\mathbb{R}^{T} = \mathbb{R} \cup \{\infty, -\infty, \Phi\}.$$

The "pictoric" representation of Transreal Numbers is usually the following one:



In the figure above, the segment of line that tends to Minus Infinity on the left and to Plus Infinity on the right represents the real numbers $\mathbb{R}$.

Some interesting statements about Transreal Numbers are the following:

1) *For all Transreal Number* $\boldsymbol{x} \neq \Phi$, $\mathbf{x} \leq \infty$;

2) *For all Transreal Numbers* $\mathbf{x} \neq \Phi$, $\mathbf{x} \geq \infty$;

3) *For all Transreal Numbers* $\mathbf{x}$, $\mathbf{x} \nless \Phi$ *and* $\mathbf{x} \ngtr \Phi$;

4) **$\infty - \infty = \Phi$;**

5) *For all Transreal Number* ***x****, the following statemants hold*:

5.1) **$x \pm \Phi = \Phi$;**

5.2) **$x.\Phi = \Phi$;**

5.3) **$x/\Phi = \Phi/x = \Phi$.**

6) *For all Transreal Numbers* **$x \neq \Phi$,** *the following statements hold:*

6.1) **$x + \infty = \infty$ ($x \neq \Phi$)**

6.2) **$x - \infty = -\infty$ ($x \neq \infty, x \neq \Phi$);**

6.3) $\infty.0 = 0.\infty = \Phi$;

6,4) $\infty/\infty = \Phi$.

In Transreal Numbers, we have infinite numbers, both positive and negative. Somehow, these numbers are already known to those who are familiar with the theory of limits within the real numbers. The fundamental difference is that, in the study of limits in the Mathematics of Real numbers, these infinities are, so to speak, "metaphors" of processes that continue indefinitely and can lead to finite numbers that are increasingly larger or smaller, as we are moving in the positive or negative directions in the infinite real line. In the domain of Transreal Numbers, minus infinity and plus infinity are *constants*, authentic and definite numbers!

In turn, in order for us to be somewhat familiar with the number "Nullity", a constant, we need to remember that such a number, since it is the result of dividing zero by zero, can be seen as the numerical expression of the undetermined, *of what is not defined by unambiguous way in the real numbers*. In this way, Nullity can be seen (an interpretation that I particularly like) as the "superposition" of all real numbers. In fact, such an interpretation is also metaphorical or imagery-like, since the concept of *superposition* of real numbers has not yet been formally defined. Thus, to say that a quantity has a value "Nullity" can be interpreted as that quantity has an undetermined value, but an indeterminacy that is not related to an epistemic order that reveals that *we do not know what the value of the quantity is*; the indeterminacy associated to a quantity that has value equal to Nullity is an objective one: such quantity takes all the possible values for that quantity at the same time in the form of a "superposition".

Let us now return to the question of why luxonic reference-frames are not authentic inertial frames in the expanded **SRT.** And to solve this question, the Transreal numbers with the "phenomenological Pythagoreanism" that will come with them will be used: what transreal numbers suggest with their infinities and with Nullity will be considered as truly descriptive of the structure of Nature and how such structure should be experienced by an observer.

Thus, in the spirit of the Phenomenological Pythagoreanism methodology, let the Transreal numbers reveal what an observer linked to a luxonic particle would experience as an instant of time, from the already presented Lorentz Transformations. By considering expression 4) above presented in Lorentz Transformation, we see that when the value of $\boldsymbol{\beta}$ is equal to **1** (luxonic case), we find an expression that has a denominator equal to zero, which implies in the Arithmetic of Transreal Numbers that the entire expression takes as value Plus Infinity. Namely:

4) $\quad \boldsymbol{t' = \mathrm{Re}\ [(t + i\tau) - u(x - i\xi)/c^2]/\sqrt{1-\beta^2}},$

if β **= 1, then:**

$t' = \mathrm{Re}\ [(t + i\tau) - u(x - i\xi)/c^2]/\sqrt{1 - 1} =$

$= \mathrm{Re}\ [(t + i\tau) - u(x - i\xi)/c^2]/0$

$= \infty$ (according to Transreal Number Arithmetic, since $= \mathrm{Re}\ [(t + i\tau) - u(x - i\xi)] > 0$ - time coordinate is always a positive real number in the reference-frame *R*)

Therefore, an observer within a luxonic particle would see each instant as being infinite. But what can be an infinite instant? Generally, an instant is seen as a point that is connected to the others through a continuous flow, and this flow is mathematically translated into a continuous segment of line. However, an infinite instant, of course, cannot be this; the infinite instant is best viewed as an "infinite line segment that" cuts "the punctual instant, extending it indefinitely; in a poetic language, we can say that the luxonic observer sees each instant as" eternal ".

Since "living" an experience of time analogous to Eternity, the notion of physical causality, based on the ordering of the instants of time, in such a way that there is precedence between them, is not sustained by the luxonic observer. Given two hypothetical events in the phenomenal field of a luxonic observer, these cannot be related as causally linked: for this to happen, the time structure of the observer “riding in a *Luxon* should allow establishing which event is previous and which one is subsequent. But this is impossible: the Luxonic observer sees all moments as infinite and, therefore, no precedence between them exists - they are *strangely* simultaneous, since they are *eternal.*

From what is given to us by the "Phenomenological Pythagoreanism" that emerges from the Transreal Numbers, we can say that the physical non-causality experienced by Luxonic observers (and is that no-causality that removes these observers from the list of authentic reference-frames) comes from the fact that the Luxonic instants are in some sense *eternal* and, therefore, are not coordinated in an order structure in which it makes sense to speak of precedent and subsequent instants; luxonic time is an "eternity" whose infinite instants are "linked to each other" by means of a strange and eternal simultaneity that embraces the *phenomenological field* of such observers that travels at speed of light…

We conclude, therefore, that the non-causality of reference-frames that move at the speed of light comes from its time structure based on eternal instants that do not maintain a relation of order between them. However, we may be curious to know how a luxonic observer would evaluate a proper time interval. For that, it is enough that we find the $\Delta t'$ corresponding to two "eternal" instants. In Transreal arithmetic, the difference between two instants t'_2 and t'_1 of values equal to infinity, results in Nullity ($\infty - \infty = \Phi$).

How should we interpret a time interval equal to Nullity, according to "Phenomenological Pythagoreanism"? It has been said earlier in this draft that a possible interpretation of Nullity is that such a number is the superposition of all real numbers – not yet a mathematical concept, but a "image". Thus, in physical contexts this *continuous* "superposition" can be interpreted in the following way: *if a physical quantity takes as its value Nullity in a given context, this means that such quantity is indeterminate in an objective sense: the reality described by such quantity is an infinite set of "positions" in space-time, given all of them simultaneously.*

In this way, a delta of time equals to Nullity means that all possible time paths between two instants are given at the same time and, therefore, measuring time in these circumstances is impossible, since measuring is "collapsing" all paths of time in a single one: the observer in a luxonic particle is immersed in a time structure in which the instants are eternal, and the path between them is a "continuous impassable block" ...

Since the delta of time of a luxonic observer is Nullity, then any velocity that such observer measures will be equal to Nullity: in Transreal arithmetic, any number divided by Nullity is equal to Nullity (Remember that speed is defined as **ds / dt**). This means that any speed measured in a luxonic reference-frame will be the “superposition” of all possible speeds in the movement from one point to another, that is, an observer traveling at the speed of light sees all things moving with nullity speed: "*continuous blocks of speeds "connecting one point to another!*

However, according the theorem of addition of velocities in the expanded **SRT** - a theorem that is deduced from Lorentz transformations (see Olkhovsky and Recami) -, the relation between the velocities observed by two reference-frame in relative motion is as follows:

$$(c^2 - v'^2)/c^2 = [(c^2 - v^2)(c^2 = u^2)/(c^2 - u.v)^2,$$

in which v is the velocity measured in the reference-frame R, v' is the velocity measured in the reference-frame R', and u is the relative velocity between R and R'.

In the above expression, when we substitute u for c, we have that v' is equal to c, regardless of the value of **v** (see in this article the conditions that relate the velocities observed by subluminal, luxonic and superluminal references). This means that an observer at a luxonic particle attributes the speed of light to any movement. However, it is worth asking whether this luminal velocity observed ubiquitously by luxonic reference-frame is the same, phenomenologically speaking, as that observed by a bradyonic observer when evaluating the photon movement. And the answer is not!

Since an observer in a luxonic reference has an indefinite proper time (equal to Nullity), any velocity that he/she measures will be equal to Nullity - the "superposition of all possible velocities of a particle". Thus, the luminal speed that the observer attributes to any movement is of a special type, since it is non-physical, this is, unrelated to the idea of physical causality, an idea that is not present in the phenomenological field of luxons. And, by this, such non-causal speed of light could be seen as something indeterminate in a physical sense, and of course the relation of such non-physical entity and Nullity is opened: *the non-causal velocity of light could be imagined as the "superposition" of all velocities* ***v*** *that are observed at subluminal and superluminal references-frames.*

The similarity between the speed of non-causal light and Nullity can be verified if we introduce a sufficient and necessary condition for a reference-frame to be a *causal reference-frame.* Namely:

A reference-frame ***R*** *is a causal reference-frame in the expanded* ***SRT*** *if, and only if, there are velocities* $\boldsymbol{v_R}$ *measured at* ***R*** *such that:*

$$(v_R < c) \quad or \quad (v_R > c).$$

By the condition above, we see that only subluminal ($\boldsymbol{R_{L-}}$) or superluminal reference-frames ($\boldsymbol{R_{L+}}$) are causal ones. In the case of luxonic observer ($\boldsymbol{R_L}$), since they only evaluate velocities as having value ***c***, we have:

For all $\boldsymbol{v}_{R_L}$, $\boldsymbol{v}_{R_L} = \boldsymbol{c}$. *Thus:*

$$(v_R \nless c) \quad \text{and} \quad (v_R \ngtr c)$$

As seen above, Nullity satisfies the following arithmetical property:

For all Transreal Numbers $\mathbf{x}$, $\mathbf{x} \nless \Phi$ *and* $\mathbf{x} \ngtr \Phi$;

Therefore, luxonic reference-frames are not causal, and within them the speed of light behaves similarly like Nullity, and this fact allows us to interpret the speed of light having a "superpositional" or metaphysical character.

Thus, from the considerations made here of what the physical world would be like if it were seen from an observer at the speed of light (and these considerations were based on the methodology of "Phenomenological Pythagoreanism" used from the Transreal Numbers), we can say poetically say the following:

From a place in the Light , everything seems eternal and mysteriously luminous ...

Some Bibliographical References:

1) Tiago S. dos Reis, Walter Gomide, James A.D.W. Anderson. **Construction of the Transreal Numbers and Algebraic Transfields.** IAENG International Journal of Applied Mathematics, 46:1, IJAM_46_1_03;
2) Paul Dirac. **The Relation between Mathematics and Physics.** Lecture delivered on presentation of the JAMES SCOTT prize, February 6, 1939. Published in: Proceedings of Royal Society (Edinburgh) Vol 59. 1938-39. Part II pp, 122-129.
3) P. Caldirola and E. Recami. **Causality and Tachyons in Relativity.** In: Italian Studies in Philosophy of Science. Boston Studies in Philosophy of Science, Volume 47, 1981.
4) V.S. Olkhovsky and E. Recami. **About Lorentz Transformations and Tachyons.** Lettere al Nuovo Cimento, Vol 1, n. 4, Genaio, 1971.